Nothing but the FACTS!

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Basic Fact Strategies

- What role does *number sense* play in learning basic facts?
- How can we think differently about supporting students so they master the basic facts?

Two Points of View

Conventional Approach

Mastery grows out of memorizing individual facts by rote through repeated practice and reinforcement.

Number Sense

Mastery that underlies computational fluency grows out of discovering the patterns and relationships that interconnect the basic facts.

Memorize this sequence of numbers

25811141720

How many digits can you recall?

“Clumping” strategy:

25811141720

258 111 417 20

Making sense….

25811141720

2 5 8 11 14 17 20

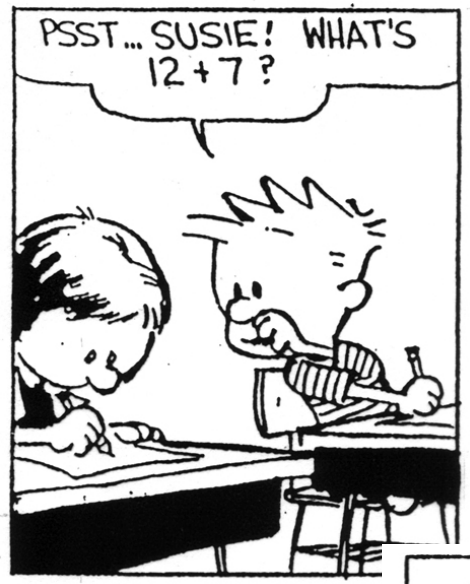
Two Points of View

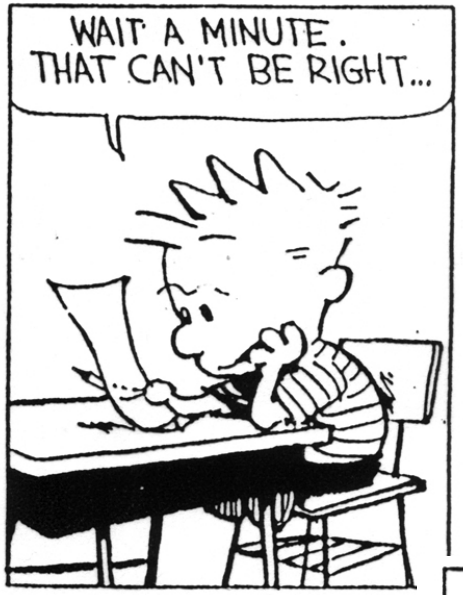
Conventional Approach

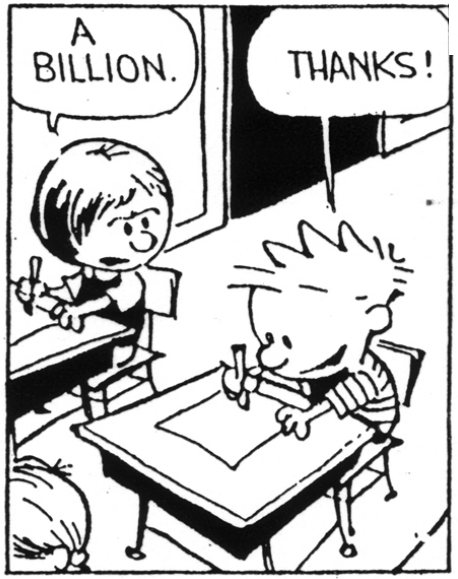
Difficulties with mastering facts are due to deficits inherent with the learner.

Number Sense

Difficulties are due to deficits inherent in conventional instruction.


PSST... SUSIE! WHAT'S 12+7?


WAIT A MINUTE. THAT CAN'T BE RIGHT...


A BILLION.
THANKS!


THAT'S WHAT SHE SAID 3+4 WAS:
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WATTERSON

- **By the end of the K-2 Program: Demonstrate fluency in addition and subtraction facts with addends through 10.**

- **By the end of the 3-4 program: Demonstrate fluency in multiplication and division facts with factors through 10.**

Big Idea #1: Composition

Numbers can be "put together" in different ways

How many ways can you line up 5 tiles using only 2 colors?

What is 9 + 8? How do you know this is true?

$$9 + 8 = 9 + 1 + 7 = 10 + 7 = 17$$

Big Idea #2: Decomposition

Numbers can be “taken apart” in different ways

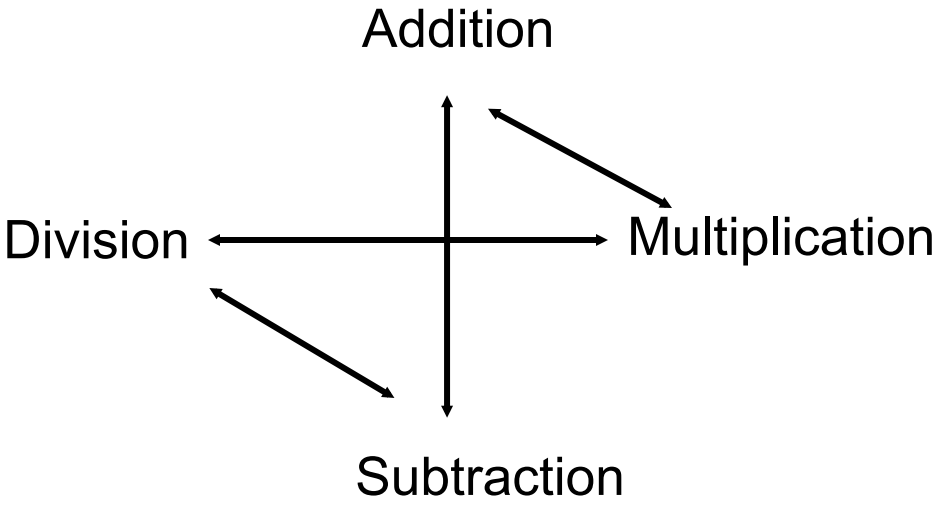
15 = 10 + 5
15 = 9 + 6
15 = 8 + 7

Big Idea # 3: The commutative property

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Big Idea #4: Relationships

- More Than
- Less Than
- Same As (Equality)



From bottom to top:

- The big base--conceptual understanding of what the operation means and number sense.
- The middle layer--strategies for learning the facts
- The top layer--the ultimate goal of instant recall. The child can answer, "What is 7 + 8?" as quickly as if asked, "What is your name?"

Developing Fluency

Phase 1 - Counting strategies

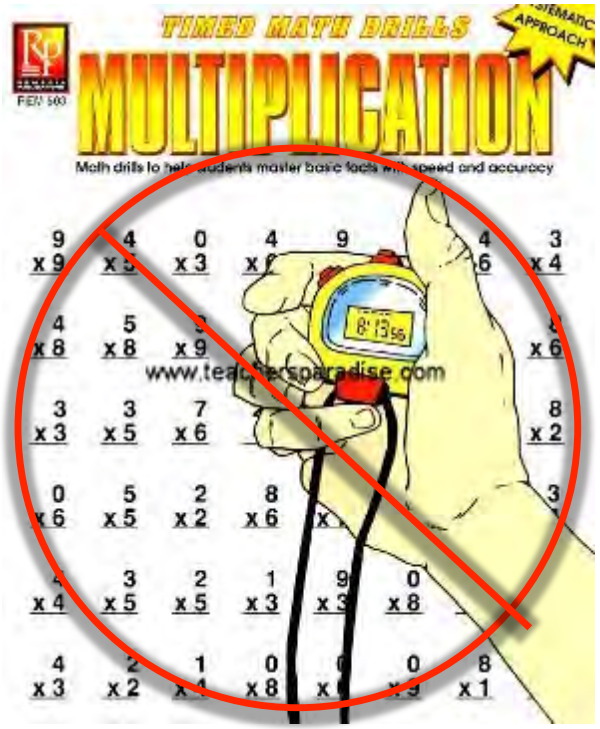
Using object counting (e.g., blocks, fingers, marks) or verbal counting to determine answer

Phase 2 - Reasoning Strategies

Using known information (e.g., known facts and relationships) to logically determine (deduce) the answer of the unknown combination

Phase 3 – Mastery

Efficient (fast and accurate) production of answers



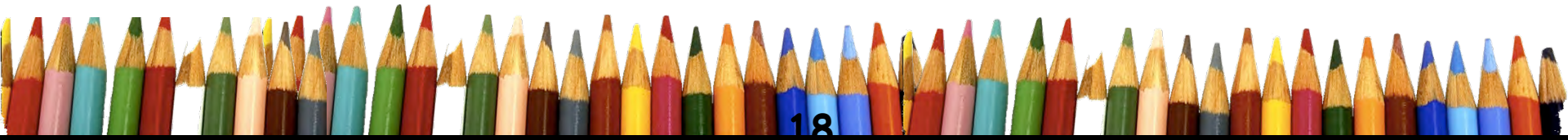
Drill of inefficient methods does NOT produce fluency!

What we know about strategies:

- An efficient strategy is one that can be done mentally and quickly. Counting is not efficient.
- The use of strategies is not new.
- You may think you "just know" the facts but it is because you have developed such efficient strategies for retrieval and they are now automatic.
- For your students to develop efficient strategies, you must have command of as many "good strategies" as possible even if you have never used them. This will help you to recognize your students' strategies.

Hints for Helping Students to Master Basic Facts

- Avoid premature drill
- Practice strategy selection/retrieval
- Make strategies explicit
- Drill established strategies
  - games
  - manipulatives
  - worded problems
- Individualize instruction



Materials that support addition and subtraction strategies

- Tens frames
- Hundreds Chart
- Linking cubes
- Quick Image Cards
- Dice
- Playing Cards
- Number Talks
- Others?

Strategies For Addition Facts

1. One-More-Than & Two-More-Than Facts
2. Facts with Zero
3. Doubles
4. Near Doubles
5. Make-Ten-Facts
6. Doubles-Plus-Two / Two-Apart Facts
7. Make Ten Extended

Strategies for Addition Facts

Near Doubles

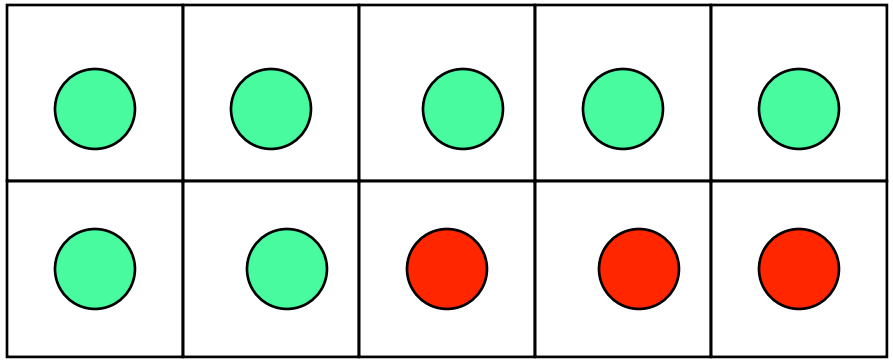
- 2 + 3 =
- 6 + 5 =
- 6 + 7 =
- 7 + 8 =

Strategies for Addition Facts

Make a Ten

- 3 + 7 =
- 6 + 4 =
- 1 + 9 =
- 8 + 2 =

Tens frames



7 + 3 = 10

Strategies for Addition Facts

"Make-Ten Extended" Strategy

7 + 4, think 7 and 3 more makes 10 and 1 left equals 11.

6 + 8, think 8 and 4 more makes 10 and 4 more makes 14.

9 + 5, think 10 + 5 and subtract 1

Number Talks

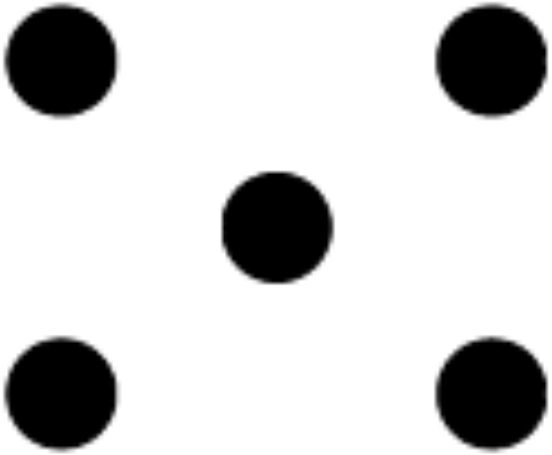
5 + 3 =

6 + 3 =

4 + 7 =

5 + 7 =

Quick Images



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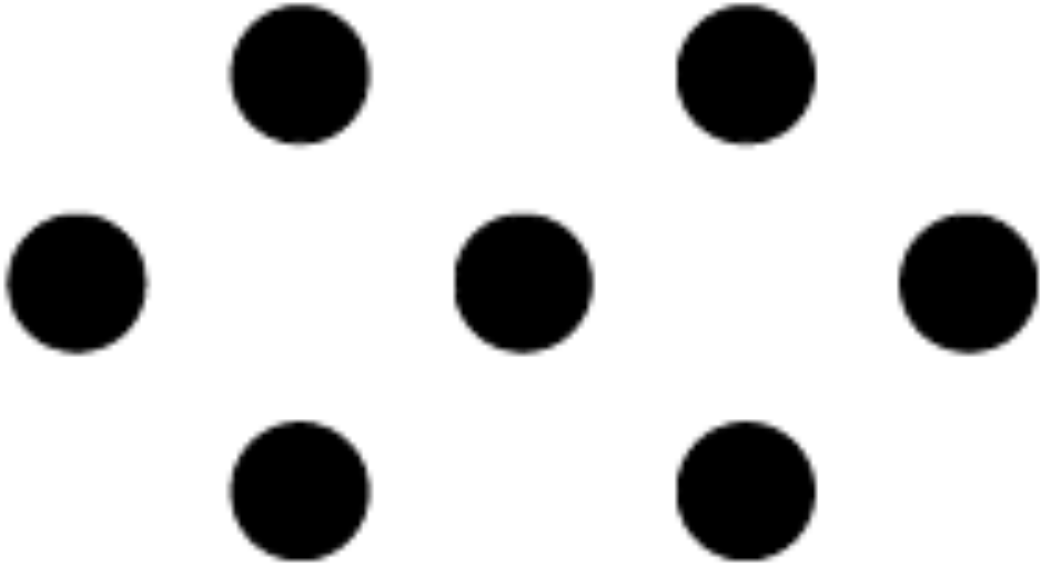

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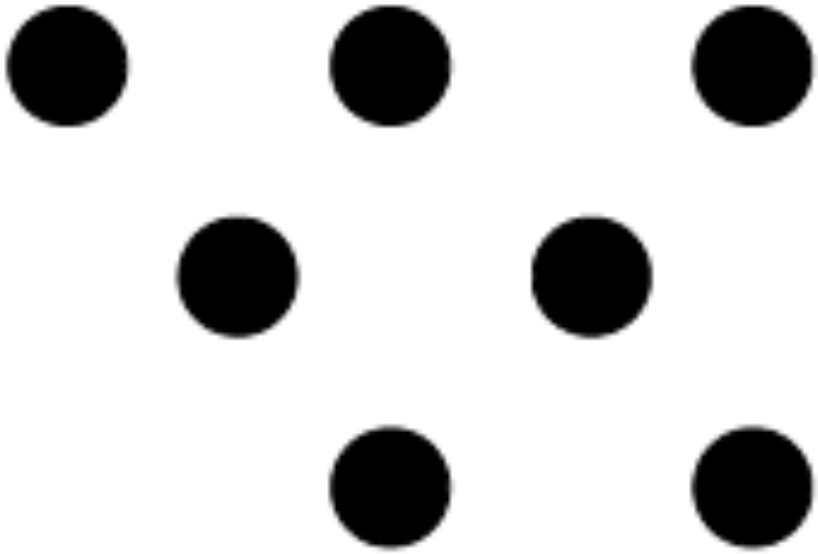
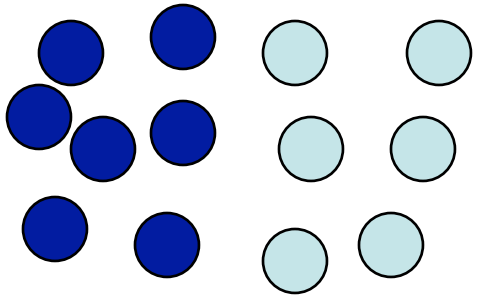


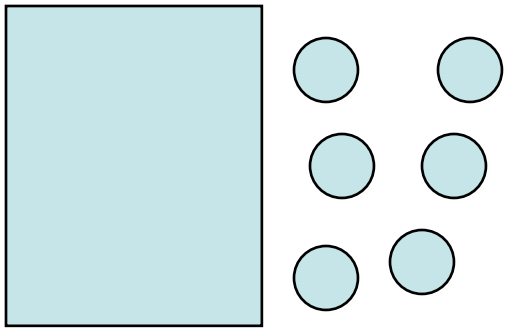
Strategies for Subtraction Facts

- Subtraction as Think-Addition
- Subtraction Facts with Sums to 10
- Build up Through 10
- Back Down Through 10
- Extend Think Addition

Think Addition



13 - 6

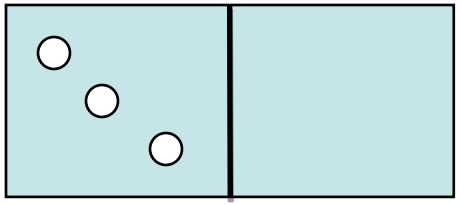


How many are under the rectangle?

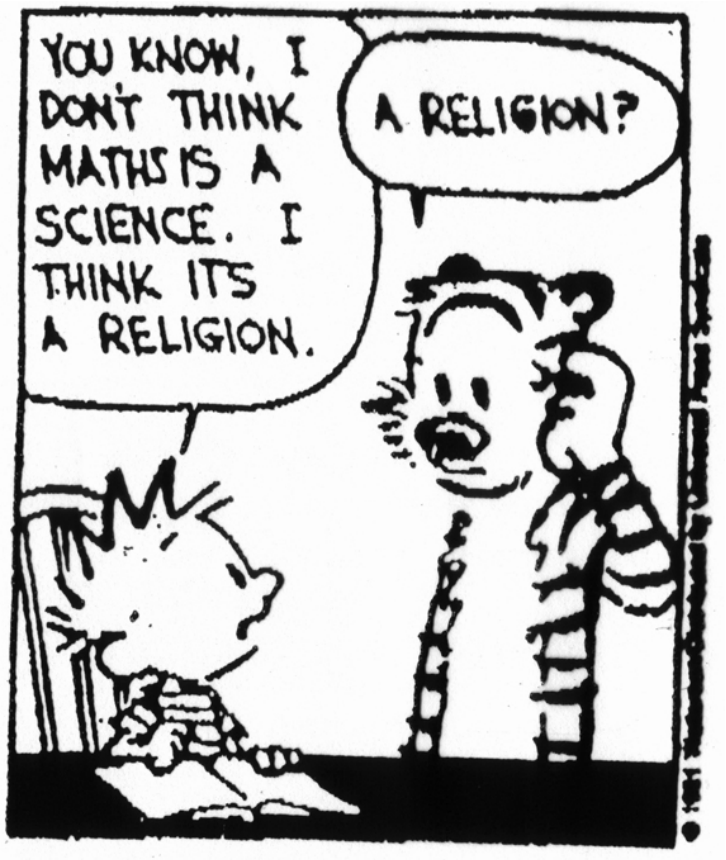
Sums to 10

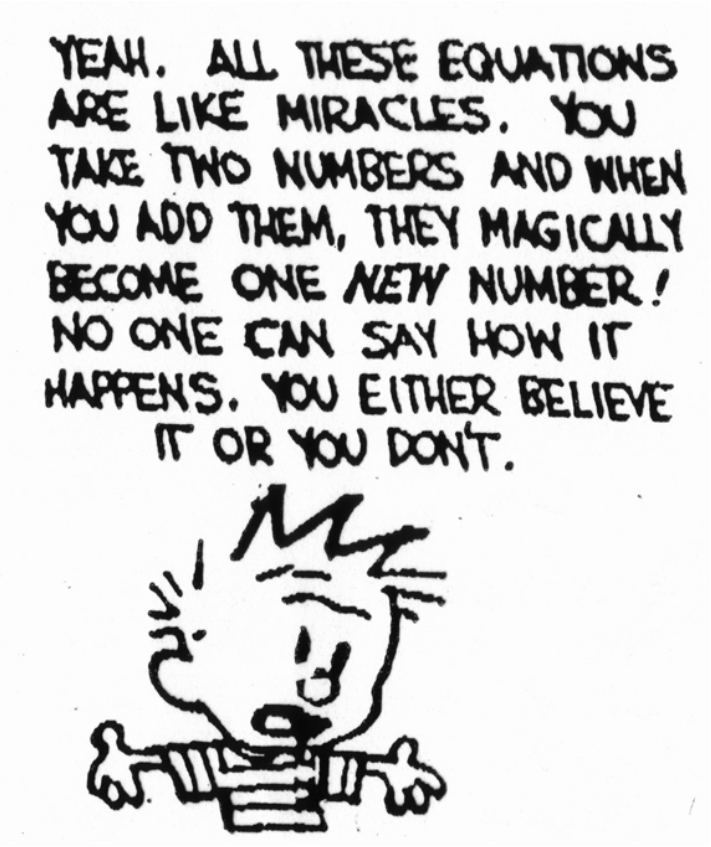
8 - 5 =

Think: 5 and how many more make 8?



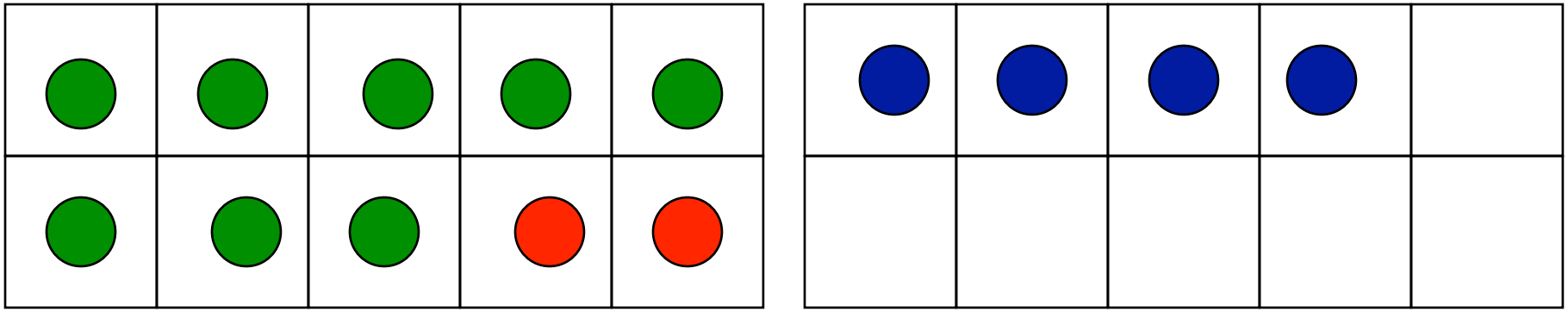
The total is 8. how many are on The other side?


YOU KNOW, I DON'T THINK MATHS IS A SCIENCE. I THINK IT'S A RELIGION.
A RELIGION?


YEAH. ALL THESE EQUATIONS ARE LIKE MIRACLES. YOU TAKE TWO NUMBERS AND WHEN YOU ADD THEM, THEY MAGICALLY BECOME ONE NEW NUMBER! NO ONE CAN SAY HOW IT HAPPENS. YOU EITHER BELIEVE IT OR YOU DON'T.

Build to 10

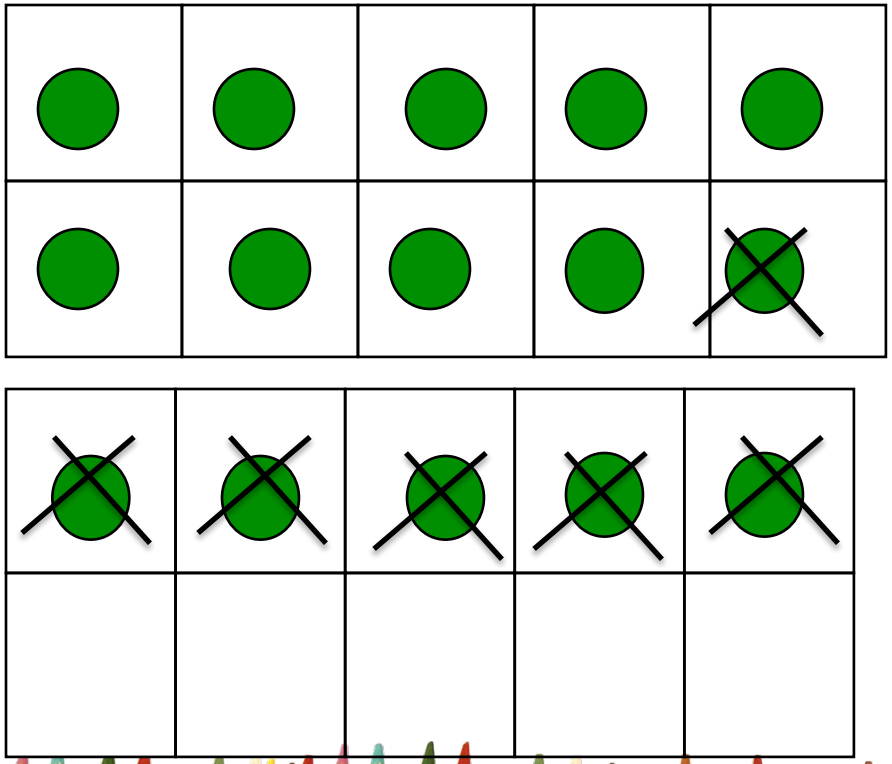
14 - 8 =



Think: Start with 8.
How many more to get to 10?
How many more to get to 14?

Back Down Through 10

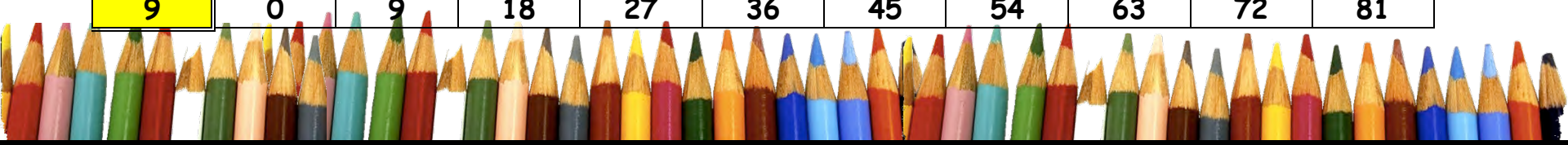
15 – 6



Take away 5 to get to 10
and 1 more leaves 9.

Multiplication Facts

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81



Multiplication Structures

What does each number represent?

$$6 \times 8 = 48$$

Multiplication Structures

- Equal Groups
  - Number of groups and size of each group is known
  - Relates to repeated addition
  - Can also be “rate” problem

Multiplication Structures

- Comparison Problems
  - One group will contain multiple copies of the other
- Combinations
  - Counting the number of pairings that can be made between 2 sets of objects
- Area and Measurement
  - Product is a different unit from the factors

Strategies for Multiplication Facts

- Doubles
- Fives
- Ones
- Zero
- Threes – triples
- Tens
- Patterns with Nines
- Fours – double double
- Eights – double double double
- Squares
- Six – triple double
- Relate to an already known fact
- Sevens

Materials to develop number sense and relationships

- Arrays
- Dot Cards/Plates
- Number Cards
- Dice
- Ten Frames
- Calculators

Multiplying by 5's

- A handful of fingers
- Counting nickels
- Using a clock
- Skip Counting
- Half of ten

Zeros and Ones

$$0 \times 7 =$$
$$1 \times 7 =$$
$$0 \times 8 =$$
$$1 \times 8 =$$

Multiplying by 1's

- Story situations
  - 1 group of 6
  - 6 groups of 1
- Use models and representations
- Avoid confusion with addition
  - 6 + 1 means 6 plus 1 more
  - 6 x 1 means 6 groups of 1

Multiplying by 0

- Story situations
  - 4 groups of 0
    - 4 chairs with no people in each chair – how many people?
  - 0 groups of 4
    - You get paid $4 for each hour you clean the yard. You spend 0 hours cleaning the yard. How much will you earn?
  - Models and representations
  - Avoid confusion with addition with 0

Multiplying by 10's

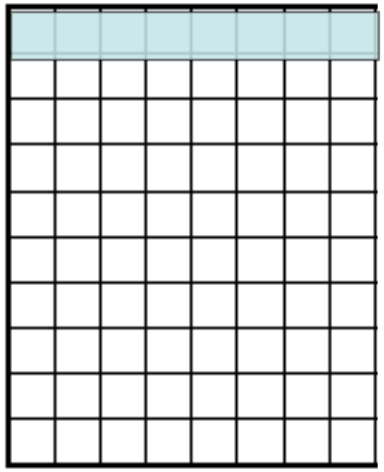
- Skip Count
- Count dimes
- Double Hands
- Look for patterns

Multiplying by 9's

- Multiply by 10 and subtract the factor

9 x 8= 10 x 8 – 8

(10)
9



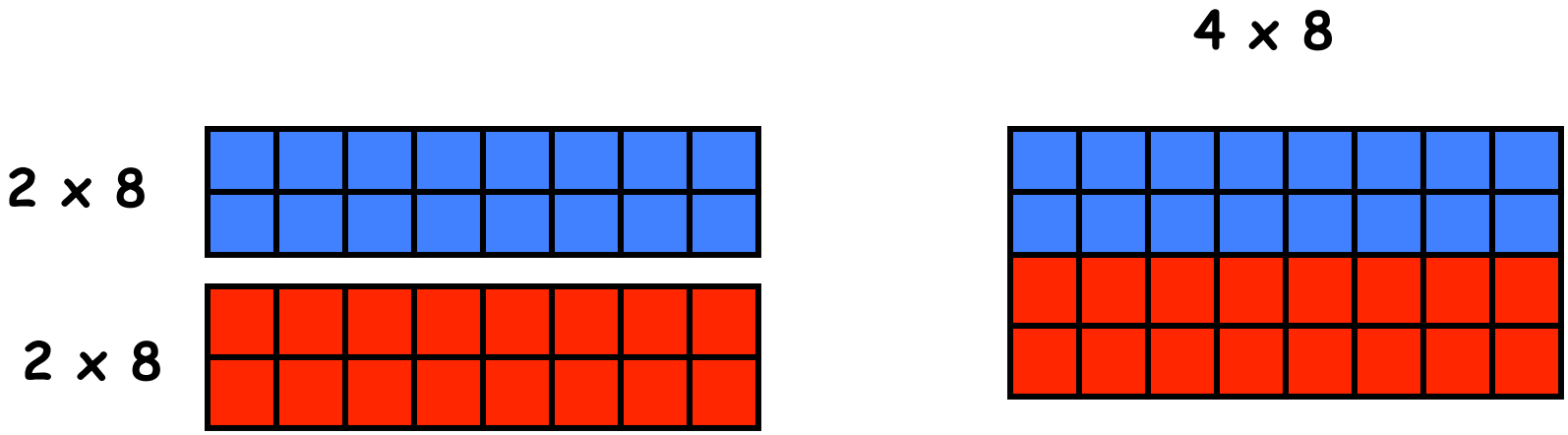
Patterns with Nines

0 x 9 = 0
1 x 9 = 9
2 x 9 =18
3 x 9 =27
4 x 9 =36

5 x 9 =45
6 x 9 =54
7 x 9 =63
8 x 9 =72
9 x 9 =81

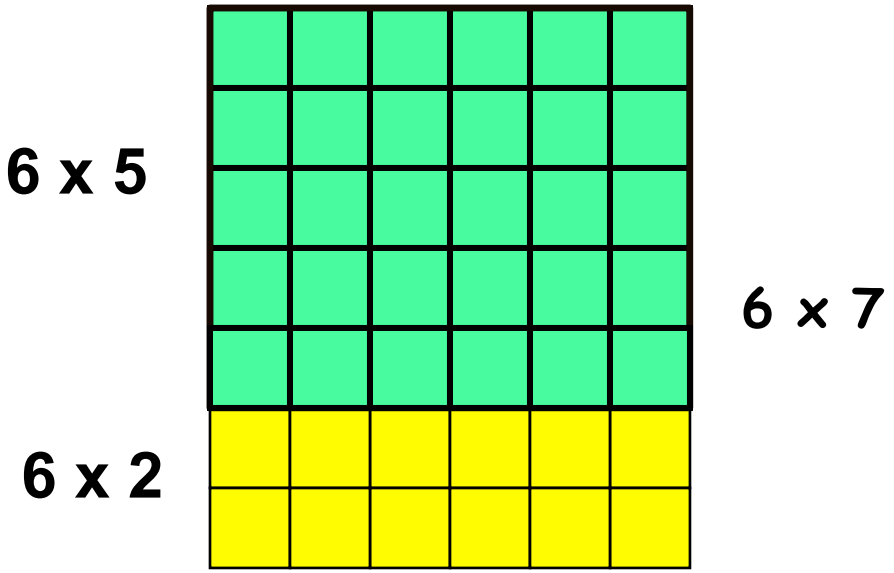
Multiplying by 4's

- Doubling doubles
  - 4 x 8 is the same as 2 x 8 doubled.



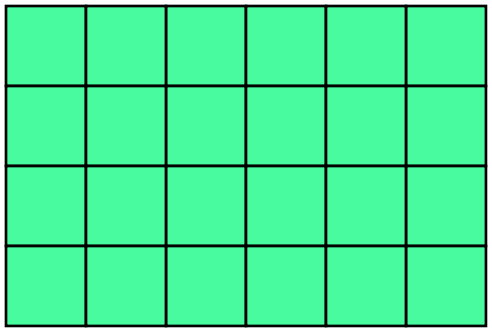
Putting Together

6 x 7 =



Halving and Doubling

6 x 4



6 x 4

3 x 8

Why does it work?

Taking apart

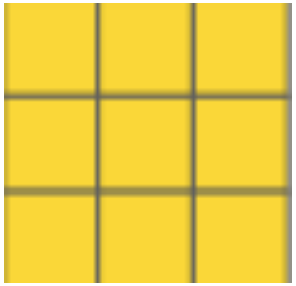
$8 \times 7 = 8 \times 8 - 8$

8 groups of 7 is the same as 8 groups of 8 minus 1 group of 8

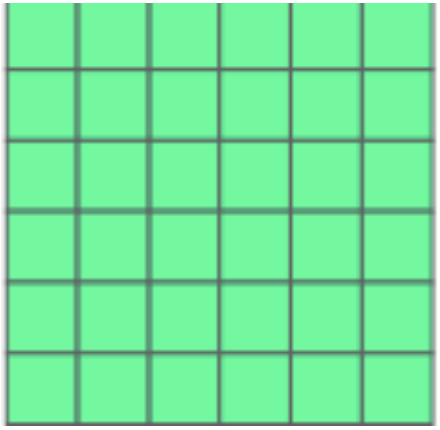
How might you model this?

Square Numbers

3 x 3 =



6 x 6 =



Relationship between multiplication and division

- In multiplication we know the number of groups and the number of items in each group and we are looking for the total number of items
- In division we know the total number of items and
  - the number in each group (looking for the number of groups) or
  - we know the number of groups (looking for the number in a group)

Relationships among operations

$$7 \times 8 = 56$$

$$56 \div 7 = 8$$

$$56 \div 8 = 7$$

The Key to Division….

- When students can ask themselves the right questions they can learn their division facts by drawing upon what they know from multiplication and using models

Asking students questions…

$8\overline{)32}$

$8\overline{)38}$

Division Facts

An interesting question to ask is “When children are working on division facts are they practicing division or multiplication?” There is undoubtedly some value in limited practice of division facts. However mastery of multiplication facts and connections between multiplication and division are the key elements of division fact mastery. Word problems continue to be a key vehicle to making this connection. (VandeWalle)

General Strategies for Division

- Count multiples
- Think multiplication
- Fact families

  $3 \times 6 = 18$ so $18 \div 3 = 6$ and $18 \div 6 = 3$
- Patterns (1, 2, 5, 9,10)
- Dividing a number by itself
- Square numbers
- Use what you know

  $56 \div 7$ if I know $49 \div 7 = 7$ then $56 \div 7 = 8$

Tool for division facts

- Arrays
- Number line
- Tiles, cubes, counters
- Triangle cards
- Missing factor cards
- Multiplication array

Effective Drill

- Drill without strategy development and number sense has repeatedly been deemed as ineffective.
- It is unreasonable to expect all children to be comfortable with the same strategies. Listen for strategies used by individuals.
- A child that has not mastered multiplication facts is not ready for division practice.

Keys to Success

- Recognize that more drill does not work!
- Inventory the known and unknown facts for each student in need.
- Diagnose strengths and weaknesses.
- Build in Success!!!!

Implementation Model

- Introduce the strategy
  - Set the stage for students to discover the strategy by strategically selecting facts
- Reinforce
- Practice
- Extend

Timed Tests???

"Teachers that use timed tests believe that the tests help the kids learn basic facts. This makes no instructional sense. Children who perform well under time pressure display their skills. Children who have difficulty with skills or who work more slowly run the risk of reinforcing wrong practices under pressure. Also they can become fearful about or negative toward their mathematical learning!"

- Marilyn Burns